

# Cross-Guide Coupler Modeling and Design

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*This report describes modeling of cross-guide couplers based on the theory of equivalent electric and magnetic dipoles of an aperture. Additional correction factors due to nonzero wall thickness and large aperture are also included in this analysis. Comparisons of the measured and calculated results are presented for cross-guide couplers with circular or cross-shaped coupling apertures. A cross-guide coupler was designed as a component of the C-band feed to support the Phobos mission.*

## I. Introduction

In this article, the performance of a directional coupler is specified by two parameters: coupling and directivity [1]. The circular aperture and cross-shaped aperture are chosen because the circular aperture is the most well-studied aperture shape and the cross-shaped aperture is frequently used for cross-guide couplers.

A program was developed to calculate the coupling and the directivity of circular or cross-shaped aperture couplers. Either one aperture (Fig. 1) or two symmetrically spaced apertures (Fig. 2) may be used for directional coupling. The angle between two waveguides is arbitrary for one-aperture couplers, but it is 90 degrees for two-aperture couplers. The apertures may be moved along the diagonal line in the common broad wall of the waveguides. Also, the distance between two apertures located on the diagonal line varies. The coupling and the directivity for directional couplers may be calculated over a band of frequencies.

## II. Theory

### A. Basic Formulas

The formulas for cross-guide couplers with off-centered apertures are derived in the Appendix. The radiation ampli-

tude in the secondary waveguide for the coupled port ( $B^+$ ) and the isolated port ( $B^-$ ) for the coupler of Fig. 1 is given by

$$B^+(p, m, d, \theta) = B_1(p, d) + B_3(m, d) \cos \theta + G(m, d) \sin \theta$$

$$B^-(p, m, d, \theta) = B_1(p, d) + B_4(m, d) \cos \theta$$

where  $\theta$  is the angle between two waveguides,  $d$  is the distance from the center of the aperture to the waveguide wall, and  $p$  and  $m$  are the electric polarizability and magnetic polarizability of the aperture.

$$p = p_0 \cdot FE \cdot TANE$$

$$m = m_0 \cdot FM \cdot TANM$$

Here,  $p_0$  and  $m_0$  are the electric polarizability and magnetic polarizability of a small aperture with zero wall thickness. They are constants that depend on the shape and the size of the aperture. For a circular aperture,  $p_0 = \frac{2}{3}r^3$ ,  $m_0 = \frac{4}{3}r^3$ , where  $r$  is the radius of the aperture. Values for  $p_0$  and  $m_0$  for a cross-shaped aperture are given in [2].  $FE$ ,  $FM$ ,  $TANE$ , and  $TANM$  are defined below.

## B. Wall Thickness Factor

The finite wall thickness of an aperture has the effect of reducing the coupling. The effect of finite thickness is taken into account by treating the aperture as a finite length of waveguide beyond cutoff.  $FE(t)$  and  $FM(t)$  represent the electric attenuation and magnetic attenuation in the aperture of thickness  $t$  and are given by [3]

$$FE(t) = e^{-2\pi \left[ \left( \frac{1}{\lambda_{c1}} \right)^2 - \frac{1}{\lambda^2} \right]^{1/2} t} \cdot AE$$

$$FM(t) = e^{-2\pi \left[ \left( \frac{1}{\lambda_{c2}} \right)^2 - \frac{1}{\lambda^2} \right]^{1/2} t} \cdot AM$$

where  $\lambda_{c1}$  = the cutoff wavelength of  $TM_{01}$  mode of aperture waveguide,  $\lambda_{c2}$  = the cutoff wavelength of  $TE_{11}$  mode of aperture waveguide,  $\lambda_{c1} = 2.6127r$ ,  $\lambda_{c2} = 3.4126r$  for a circular waveguide, while  $\lambda_{c1}$  and  $\lambda_{c2}$  of a cross-shaped waveguide are given in [4] and [5].

The additional factors  $AE$  and  $AM$  are effective wall thickness coefficients. For a circular aperture with radius  $r$  and thickness  $t$  [6], [7]

$$AE = 1.0103 + 0.0579 \frac{r}{t} \quad t/r > 0.2$$

$$AM = 1.0064 + 0.0819 \frac{r}{t}$$

$$AE = 1.1091 - 0.0082268 \frac{r}{t} \quad t/r \leq 0.2$$

$$AM = 1.4273 - 0.0023284 \frac{r}{t}$$

The  $AE$  and  $AM$  of a cross-shaped aperture are determined experimentally.

## C. Large Aperture Factor

An infinitely thin aperture actually has an unlimited number of resonances. For a large aperture, the following frequency correction factors are needed [8]:

$$TANE = \frac{2f_{01}}{\pi f} \tan \frac{\pi f}{2f_{01}}$$

$$TANM = \frac{2f_{02}}{\pi f} \tan \frac{\pi f}{2f_{02}}$$

The resonant frequency is approximately equal to the cutoff frequency of a waveguide having the same cross-sectional shape and size as the aperture. Therefore,  $f_{01}$  may be replaced by  $f_{c1}$  and  $f_{02}$  may be replaced by  $f_{c2}$ .

$$f_{c1} = \frac{c}{\lambda_{c1}}$$

$$f_{c2} = \frac{c}{\lambda_{c2}}$$

## III. Results

### A. One-Circular-Aperture Coupler with $\theta = 45$ Degrees

An off-centered circular-aperture coupler with adjustable  $\theta$  has been fabricated. The dimensions of the WR112 coupler are 0.17-inch aperture radius, 0.128-inch thickness, and 0.283-inch distance from center of aperture to waveguide wall. For  $\theta = 45$  degrees, the calculated and the measured coupling and directivity at frequencies from 7 to 9 GHz are shown in Figs. 3 and 4, respectively. The calculated coupling is 0.5 to 0.7 dB higher and the directivity is 0.9 to 1.1 dB lower than measured. The results show good agreement between the coupler model and the experiment.

### B. Two-Circular-Aperture Cross-Guide Coupler

A two-aperture cross-guide coupler using circular apertures was designed based on the preceding theory. The final coupler design had the following dimensions: WR125, 0.13-inch aperture radius, 0.05-inch thickness, and 0.3125-inch distance from center of aperture to waveguide wall. The measured coupling is 0.2 to 0.6 dB lower than calculated, while the directivity is 1.0 to 1.4 dB higher than expected at frequencies from 7 to 9 GHz (Figs. 5 and 6). In this case, and in general, the coupling is predicted more accurately than the directivity. The coupler computer program provides a pessimistic value of directivity.

### C. Two-Cross-Shaped-Aperture Cross-Guide Coupler

As an example of a design using cross-shaped apertures, a C-band coupler meeting the following requirements was designed.

Coupling:  $-30 \pm 1$  dB

Directivity: 20 dB minimum

Waveguide: WR187

Frequency: 4.96–5.06 GHz

The final design uses two symmetric cross-shaped apertures of 0.662-inch length, 0.115-inch width, 0.05-inch thickness, and located at a 0.468-inch distance from center of aperture to waveguide wall (Fig. 7).  $AE = 1.36$  and  $AM = 1.53$  were determined from a previous WR125 30-dB coupler experiment. The coupler is expected to have approximately -30-dB coupling and 25.6-dB directivity according to the coupler computer program.

The measured and computed results are shown in Figs. 8 and 9. In this case, the experimental coupling is approximately 0.4 dB lower and the directivity is 0.8 to 1.5 dB higher than

calculated. Figures 8 and 9 show that the directional coupler meets the design requirements.

#### IV. Conclusion

A brief description of a cross-guide coupler model was presented. Three specific examples have demonstrated good agreement between experiment and theory. In most cases, a suitable coupler can be designed using the simple theory presented in this report. Further directional coupler study is required to improve the prediction of directivity. Coupling between apertures and the nonuniform field over the aperture could be included in the model to obtain higher accuracy.

### Acknowledgment

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### References

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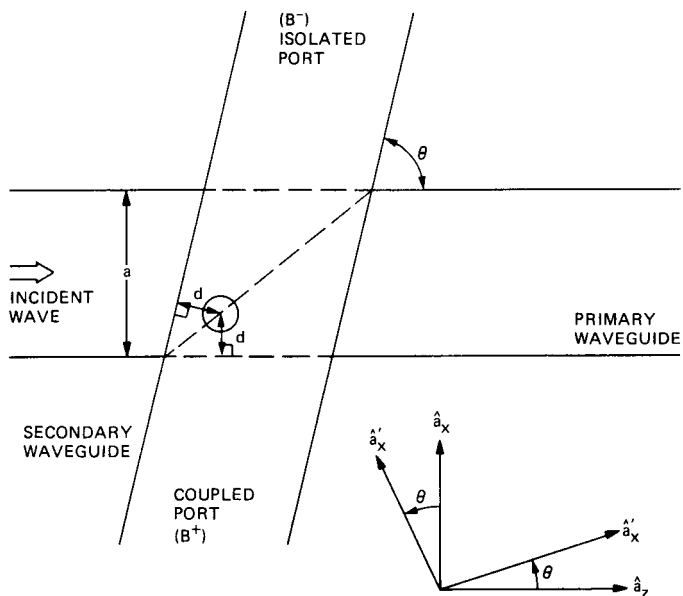


Fig. 1. Configuration of single-aperture coupler.

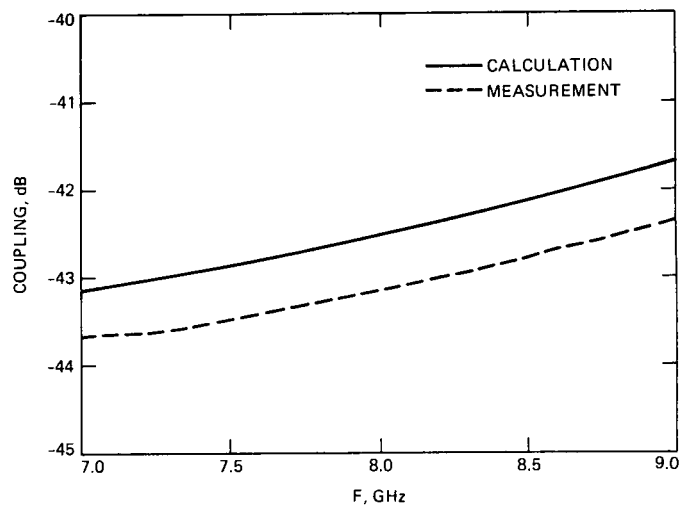


Fig. 3. Coupling of single-circular-aperture coupler.

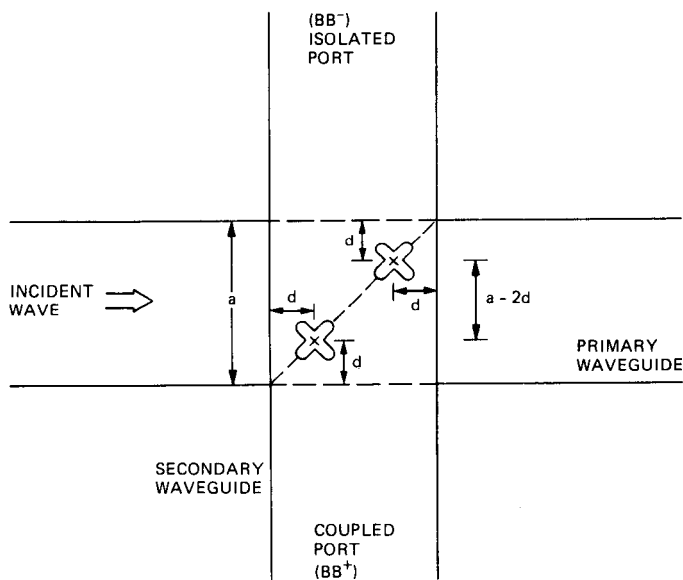


Fig. 2. Configuration of two-aperture cross-guide coupler.

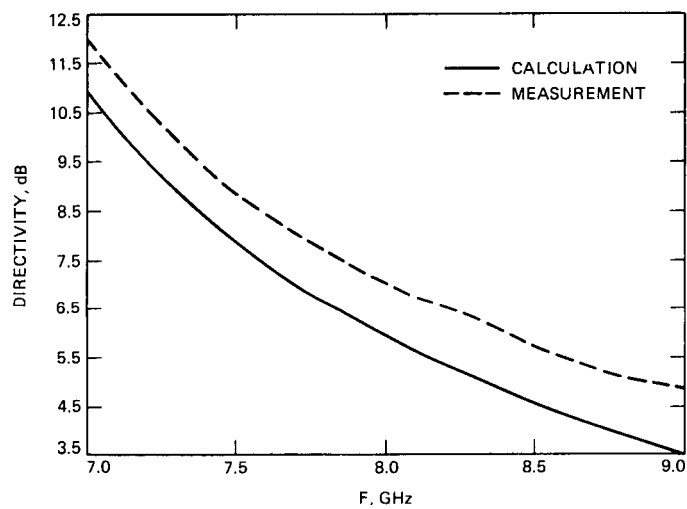


Fig. 4. Directivity of single-circular-aperture coupler.

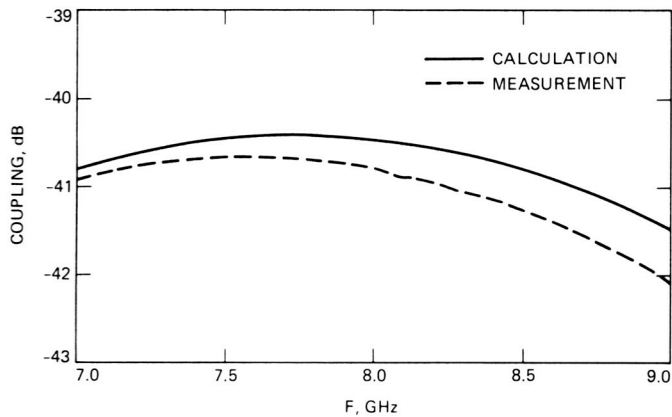


Fig. 5. Coupling of two-circular-aperture cross-guide coupler.

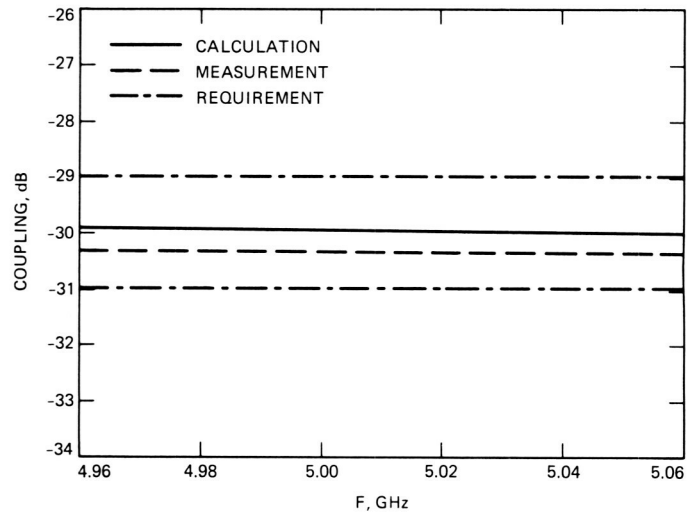


Fig. 8. Coupling of two-cross-shaped-aperture cross-guide coupler.

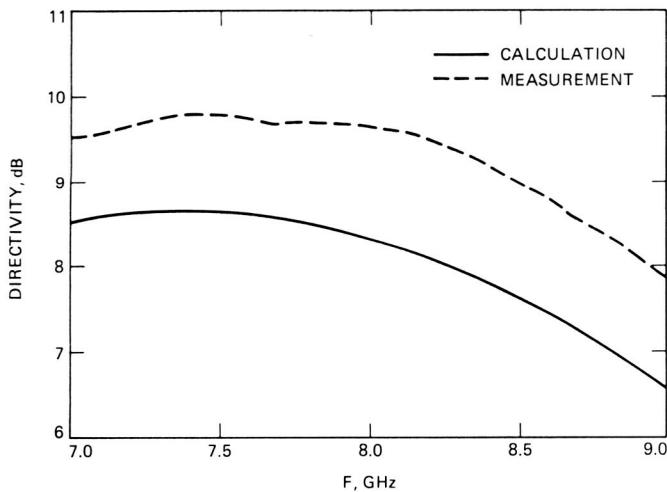


Fig. 6. Directivity of two-circular-aperture cross-guide coupler.

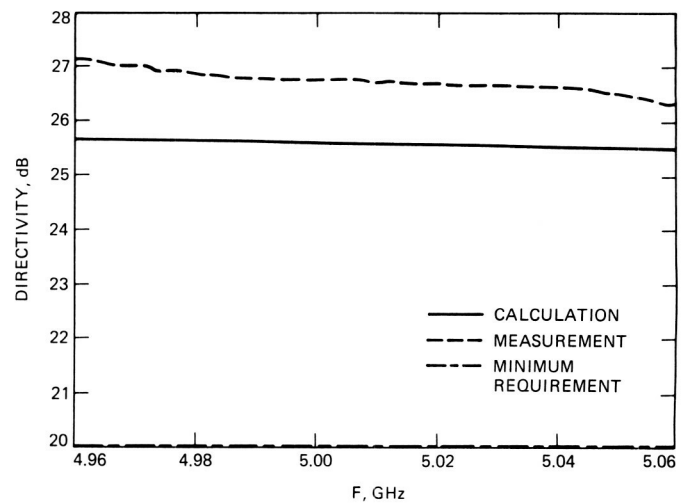


Fig. 9. Directivity of two-cross-shaped-aperture cross-guide coupler.

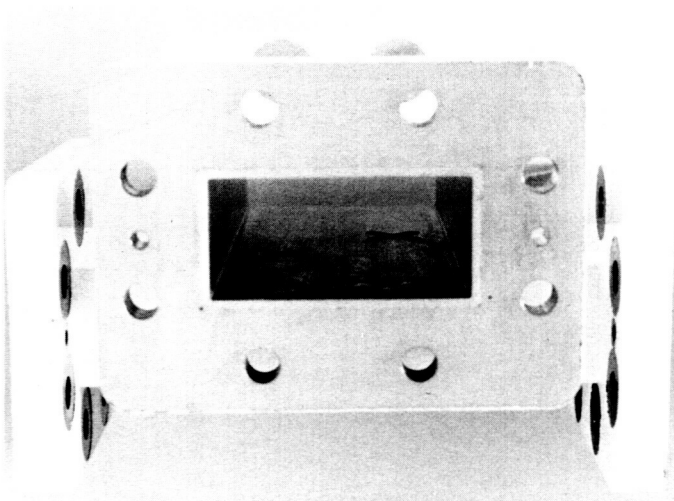


Fig. 7. A C-band two-cross-shaped-aperture cross-guide coupler.

## Appendix

### Equations for Offset Aperture with an Arbitrary Angle $\theta$

In this appendix, the results in [1] are extended to include an offset aperture with an arbitrary angle  $\theta$  between two waveguides.

Referring to Figs. 1 and 2, assume the dimensions of the rectangular waveguides are  $a$  and  $b$ . The incident field in the primary waveguide is  $TE_{10}$  with amplitude  $A$ .

#### I. One-Offset Aperture with Arbitrary $\theta$

The equivalent electric and magnetic dipoles ( $\bar{P}$ ,  $\bar{M}$ ) located at the center of the aperture ( $z = 0$ ,  $x = d$ ) for radiation into the secondary waveguide are

$$\bar{P} = \epsilon_0 p A \sin \frac{\pi d}{a} \hat{a}_y \quad (\text{A-1})$$

$$\bar{M} = -m A Y_\omega \left( -\sin \frac{\pi d}{a} \hat{a}_x + j \frac{\pi}{\beta a} \cos \frac{\pi d}{a} \hat{a}_z \right) \quad (\text{A-2})$$

where  $p$  and  $m$  are the electric and magnetic polarizability,  $Y_\omega$  is the wave admittance for  $TE_{10}$  mode, and  $\beta$  is the propagation constant.

If the secondary waveguide with a coordinate system defined by unit vectors  $\hat{a}'_x, \hat{a}'_y, \hat{a}'_z$  is rotated by an angle  $\theta$  with respect to the primary waveguide with a coordinate system defined by unit vectors  $\hat{a}_x, \hat{a}_y, \hat{a}_z$  (see Fig. 1)

$$\hat{a}_x = \cos \theta \hat{a}'_x + \sin \theta \hat{a}'_z \quad (\text{A-3})$$

$$\hat{a}_y = \hat{a}'_y \quad (\text{A-4})$$

$$\hat{a}_z = -\sin \theta \hat{a}'_x + \cos \theta \hat{a}'_z \quad (\text{A-5})$$

Substituting Eqs. (A-3), (A-4), and (A-5) into Eq. (A-1) and (A-2)

$$\bar{P}' = \epsilon_0 p A \sin \frac{\pi d}{a} \hat{a}'_y$$

$$\begin{aligned} \bar{M}' = -m A Y_\omega \left[ -\left( \sin \frac{\pi d}{a} \cos \theta + j \frac{\pi}{\beta a} \cos \frac{\pi d}{a} \sin \theta \right) \hat{a}'_x \right. \\ \left. + \left( -\sin \frac{\pi d}{a} \sin \theta + j \frac{\pi}{\beta a} \cos \frac{\pi d}{a} \cos \theta \right) \hat{a}'_z \right] \end{aligned}$$

The field radiated by the electric dipole has an amplitude

$$\begin{aligned} B_1(p, d, \theta) &= \frac{j\omega}{abY_\omega} \bar{E}_{10}^- \cdot \bar{P}' \\ &= -\frac{j\omega\epsilon_0}{abY_\omega} p A \sin^2 \frac{\pi d}{a} \\ &= B_1(p, d) \end{aligned}$$

in the coupled port and

$$\begin{aligned} B_2(p, d, \theta) &= \frac{j\omega}{abY_\omega} \bar{E}_{10}^{+'} \cdot \bar{P}' \\ &= B_1(p, d) \end{aligned}$$

in the isolated port.

The field radiated by the magnetic dipole has amplitude

$$\begin{aligned} B_3(m, d, \theta) &= \frac{j\omega}{abY_\omega} \bar{B}_{10}^- \cdot \bar{M}' \\ &= \frac{j\omega\mu_0 Y_\omega}{ab} m A \left[ \left( \sin^2 \frac{\pi d}{a} + \frac{\pi^2}{\beta^2 a^2} \cos^2 \frac{\pi d}{a} \right) \cos \theta \right. \\ &\quad \left. + 2j \frac{\pi}{\beta a} \sin \frac{\pi d}{a} \cos \frac{\pi d}{a} \sin \theta \right] \\ &= B_3(m, d) \cos \theta + G(m, d) \sin \theta \end{aligned}$$

in the coupled port and

$$\begin{aligned} B_4(m, d, \theta) &= \frac{j\omega}{abY_\omega} \bar{B}_{10}^{+'} \cdot \bar{M}' \\ &= \frac{j\omega\mu_0 Y_\omega}{ab} m A \left( -\sin^2 \frac{\pi d}{a} + \frac{\pi^2}{\beta^2 a^2} \cos^2 \frac{\pi d}{a} \right) \cos \theta \\ &= B_4(m, d) \cos \theta \end{aligned}$$

in the isolated port.

Therefore, the field in the secondary waveguide has an amplitude

$$B^+(p, m, d, \theta) = B_1(p, d) + B_3(m, d) \cos \theta + G(m, d) \sin \theta$$

in the coupled port and

$$B^-(p, m, d, \theta) = B_1(p, d) + B_4(m, d) \cos \theta$$

in the isolated port.

The coupling ( $C$ ) and directivity ( $D$ ) are given

$$C = 20 \log \left( \frac{B^+(p, m, d, \theta)}{A} \right)$$

$$D = 20 \log \left( \frac{B^+(p, m, d, \theta)}{B^-(p, m, d, \theta)} \right)$$

Some special cases are

$$(A) \quad d = \frac{a}{2}, \theta \neq 90^\circ$$

$$G\left(m, \frac{a}{2}\right) = 0$$

$$B_3\left(m, \frac{a}{2}\right) = -B_4\left(m, \frac{a}{2}\right)$$

$$B^+\left(p, m, \frac{a}{2}, \theta\right) = B_1\left(p, \frac{a}{2}\right) + B_3\left(m, \frac{a}{2}\right) \cos \theta$$

$$B^-\left(p, m, \frac{a}{2}, \theta\right) = B_1\left(p, \frac{a}{2}\right) - B_3\left(m, \frac{a}{2}\right) \cos \theta$$

$$(B) \quad d = \frac{a}{2}, \theta = 90^\circ$$

$$B^+\left(p, m, \frac{a}{2}, 90^\circ\right) = B^-\left(p, m, \frac{a}{2}, 90^\circ\right) = B_1\left(p, \frac{a}{2}\right)$$

$$D = 0 \text{ dB}$$

$$(C) \quad d \neq \frac{a}{2}, \theta = 0^\circ$$

$$B^+(p, m, d, 0^\circ) = B_1(p, d) + B_3(m, d)$$

$$B^-(p, m, d, 0^\circ) = B_1(p, d) + B_4(m, d)$$

$$(D) \quad d \neq \frac{a}{2}, \theta = 90^\circ$$

$$B^+(p, m, d, 90^\circ) = B_1(p, d) + G(m, d)$$

$$B^-(p, m, d, 90^\circ) = B_1(p, d)$$

## II. Two Symmetrically Spaced Apertures with $\theta = 90^\circ$ Degrees

For two symmetrically spaced apertures which are located at  $d_1 = d$  and  $d_2 = a - d$ , let  $\Delta = a - 2d$ ,  $\theta = 90^\circ$  (Fig. 2). The field radiated in the secondary waveguide is

$$BB^+(p, m, d, 90^\circ) = B^+(p, m, d, 90^\circ) + B^+(p, m, a - d, 90^\circ) e^{-2j\beta\Delta}$$

in the coupled port and

$$BB^-(p, m, d, 90^\circ) = [B^-(p, m, d, 90^\circ) + B^-(p, m, a - d, 90^\circ)] e^{-j\beta\Delta}$$

in the isolated port.